COUPLED HEAT TRANSFER IN NONSTEADY FLOW ABOUT A ROD BUNDLE

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The finite elements method and finite differences method are used together to study a nonsteady three-dimensional coupled problem of convective heat transfer during longitudinal flow about a bundle of heat-emitting rods.

Nonsteady coupled heat-transfer problems are of practical interest in connection with problems of regulating and controlling heat exchangers operating under heavy thermal loads and designing exchangers in which the supply of heat-transfer agent is unsteady.

Let laminar steady hydrodynamically stabilized flow of a viscous incompressible fluid with a mean velocity $\bar{w}_z$ occur in a rod bundle with the volume rate of heat release

$$Q_1 = q_v/\bar{q}_v = 1{,}25\cos[0{,}722(2z-1)\pi/2] \tag{1}$$

the flow being due to a constant pressure gradient. We assume the thermophysical properties of the fluid, rods, and shells to be constant. The fluid has a constant temperature T_0 at the inlet of the channel. The wall of the channel is thermally insulated. Internal heat release in the system results in the formation of a stationary temperature field. At the moment of time $\tau = 0^+$, pulations are superimposed on the stationary pressure gradient. The pulsations are such that the instantaneous pressure gradient is a periodic function of time:

$$\partial p/\partial z = (dp/dz)_s(1+\gamma f(\omega\tau)). \tag{2}$$

Here, the velocity vector of the fluid remains parallel to the tube axis z, i.e., $w_x = w_y = 0$, and it follows from the continuity equation that $\partial w_z/\partial z = 0$.

The following equations are included in the boundary-value problem of coupled heat convection in a bundle of fuel elements for the functions $\Theta_i = \lambda_3(T_i - T_0)/\bar{q}_v d^2$ (i = 1, 2, 3):

the energy equation

$$K_{c_i}\frac{\partial\Theta_i}{\partial \mathrm{Fo}} + \mathrm{Pe}\,LW\frac{\partial\Theta_i}{\partial Z} = K_{\lambda_i}\left(\frac{\partial^2\Theta_i}{\partial X^2}+\frac{\partial^2\Theta_i}{\partial Y^2}+L^2\frac{\partial^2\Theta_i}{\partial Z^2}\right) + Q_i(X, Y, Z, \mathrm{Fo}) \tag{3}$$

and the equation of motion

$$\frac{\partial W}{\partial \mathrm{Fo}} = \mathrm{Pr}\left(\frac{\partial^2 W}{\partial X^2}+\frac{\partial^2 W}{\partial Y^2}\right)+\frac{1}{2}\frac{d}{d_e}\xi\,\mathrm{Re}\,\mathrm{Pr}[1+\gamma f(M\,\mathrm{Pr}\,\mathrm{Fo})] \quad (\mathrm{Fo}>0;\ X, Y\in\Omega_3). \tag{4}$$

The velocity function W is equal to zero in the regions Ω_1 and Ω_2. The function Q_1 is determined from Eq. (1), while $Q_2 = 0$ and $Q_3 = \mathrm{Br}[(\partial W/\partial X)^2 + (\partial W/\partial Y)^2]$.

As the initial conditions for temperature, we chose the solution of the corresponding steady-state problem with a steady fluid flow

$$\Theta_i(X, Y, Z, 0) = \Theta_{is}(X, Y, Z), \quad i = 1, 2, 3, \tag{5}$$

while the initial conditions for velocity consists of the velocity profile of a hydrodynamically stabilized steady flow:

$$W(X, Y, 0) = W_s(X, Y). \tag{6}$$

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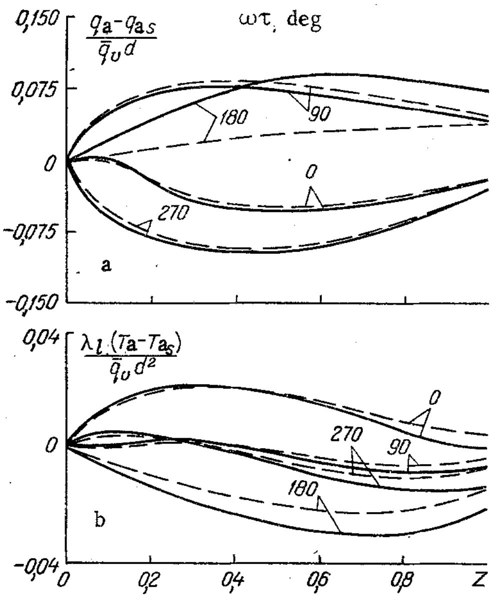

Fig. 1

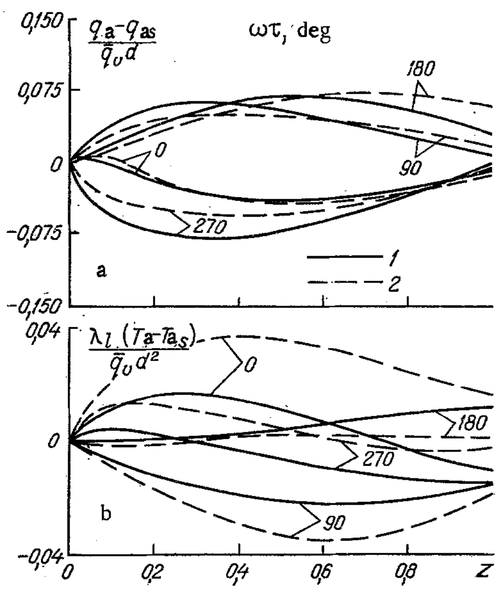

Fig. 2

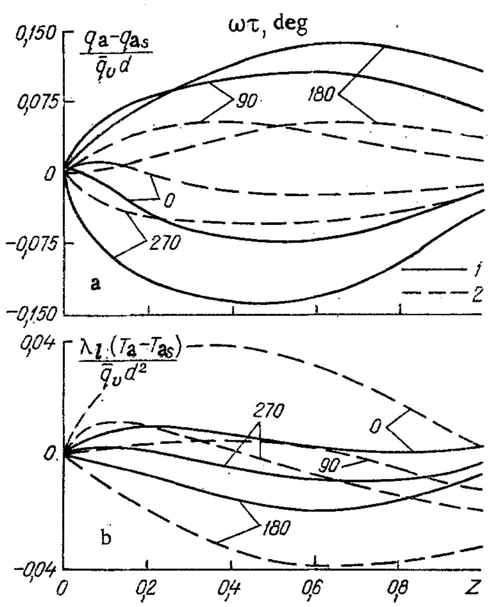

Fig. 3

Fig. 1. Difference between unsteady and steady values of heat flux (a) and temperature (b), averaged over the surface of the rod casing, for the first (solid lines) and second (dashed lines) variants of pulsating flow.

Fig. 2. Effect of the thermal conductivity of the components of a translational element of a bundle of fuel elements on heat transfer: 1) $K_{\lambda_1} = 7.9$; $K_{\lambda_2} = 30.1$; $K_{\lambda_3} = 1$; 2) $K_{\lambda_1} = 0.158$; $K_{\lambda_2} = 0.6$; $K_{\lambda_2} = 1$.

Fig. 3. Effect of the volumetric heat capacity of the components of a translational element of a fuel-element bundle on heat transfer: 1) $K_{c_1} = 2.13$; $K_{c_2} = 1.8$; $K_{c_3} = 1$; 2) $K_{c_1} = 0.24$; $K_{c_2} = 0.2$; $K_{c_3} = 1$.

The functions Θ_{is} and W_s were determined by the finite elements method [1]. The following conditions were assigned at the inlet and outlet of the heat-transfer section

$$\partial\Theta_1/\partial Z = \partial\Theta_2/\partial Z = 0, \quad \Theta_3 = 0 \quad \text{for} \quad Z = 0, \tag{7}$$

$$\partial\Theta_1/\partial Z = \partial\Theta_2/\partial Z = \partial\Theta_3/\partial Z = 0 \quad \text{for} \quad Z = 1. \tag{8}$$

Conditions of ideal thermal contact were assigned on the surface washed by the fluid and the cylinder-shell contact surface.

The complexity of studying the process described by boundary-value problem (1)-(8) stems from the need to solve a system of partial differential equations in a region of complex form. Among the studies of coupled heat transfer in the case of longitudinal flow about heat-emitting rods, we should point out the monograph [2]. The authors of [2] employed electrical modeling to calculate the steady-state temperature fields in a system comprised of a heat-emitting rod and a coolant. Steady-state coupled problems in channels of complex form were studied by approximate analytical methods in [3] and by the finite elements method in [4]. Unsteady coupled heat transfer was studied in channels of simple form by numerical and experimental methods in [5-7].

To study unsteady coupled heat transfer in longitudinal steady hydrodynamically-stabilized flow about a bundle of heat-emitting rods in [8], the authors proposed a quite laborious method based on the principle of superposition for linear problems. In [9, 10], we tried using a more universal approach involving joint application of the finite elements method for the elliptical coordinates of the channel cross section and the finite differences method for the parabolic coordinates (time and the axial coordinate) to solve unsteady coupled problems of convective heat transfer in tubes of complex form with a hydrodynamically-stabilized flow of fluids. No previous investigations have been made of a coupled heat-transfer problem involving unsteady flow about a rod bundle with nonuniform heat release along the channel using the formulation (1)-(8).

To solve problem (1)-(8), we used both the finite elements method and the finite differences method [9]. The two methods were incorporated into a single application package [10]. The effect of the unsteadiness of the flow on heat transfer was studied with two laws of change in the pressure gradient:

$$f(\omega\tau)=\begin{cases}\sin\omega\tau & \text{at}\quad 0\leqslant\omega\tau<\pi/2 \text{ and } \dfrac{3}{2}\pi\leqslant\omega\tau\leqslant 2\pi,\\ 1 & \text{at}\quad \pi/2\leqslant\omega\tau<\pi,\\ -1 & \text{at}\quad \pi\leqslant\omega\tau<3/2\pi;\end{cases} \tag{9}$$

$$f(\omega\tau)=\sin\omega\tau \quad \text{at} \quad 0\leqslant\omega\tau\leqslant 2\pi. \tag{10}$$

In calculations of the cross section of the channel, we used 840 triangular elements with 462 nodes. We used the iterative difference scheme in [11] for the variables Fo and Z. Also, we assumed that Pr = 1; γ = 0.9; Re = 200; L = 0.02; δ/d = 0.06; Br = 0; M = 2.7; the packing-density parameter of the bundle β = 1.2. Figure 1, with K_{λ_1} = 7.9; K_{λ_2} = 30.1; K_{λ_3} = 1; K_{c_1} = 0.71; K_{c_2} = 0.6; K_{c_3} = 1, shows the difference between the instantaneous unsteady and steady values of heat flux (Fig. 1a) and temperature (1b) on the rod surface for different flow variants with the same mean (over time and the cross section) velocity $\overline{w}_z$, determined by Eqs. (9) and (10). It can be seen from Fig. 1 that the amount of deviation of the theoretical characteristics of unsteady heat transfer from the steady-state characteristic changes not only with respect to time, but also the length of the channel. This can be attributed to the nonuniformity of the heat release. It is also evident from Fig. 1 that a change in the character of flow also causes substantial deviations of the heat-transfer characteristics from the steady-state values. These deviations are more significant with regard to heat flux than temperature. Figure 2 illustrates the effect of a change in the heat-conducting properties of a translational element on the difference between the values of heat flux (a) and temperature (b) averaged about the perimeter of a heat-emitting element with pulsating [in accordance with (9)] and steady flow of the coolant at the same mean (with respect to time and the cross section) velocity $\overline{w}_z$ (K_{c_1} = 0.71; K_{c_2} = 0.6; K_{c_3} = 1). It follows from Fig. 2 that a change in the thermal conductivities of the rod material and coolant has a greater effect on the temperature distribution along the channel than on the change in heat flux. Figure 3 shows the effect of the volumetric heat capacity of the components of a translational element of a bundle of heat-emitting rods on the difference between the values of heat flux (a) and temperature (b) averaged about the perimeter of the element with pulsating [in accordance with (9)] and steady-state flow of the fluid at the mean velocity $\overline{w}_z$ (K_{λ_1} = 7.9; K_{λ_2} = 30.1; K_{λ_3} = 1). It is evident from Fig. 3 that, in contrast to a change in thermal conductivity, a change in volumetric heat capacity causes substantial deviations in the distribution of both heat flux and temperature. Meanwhile, an increase in the heat capacity of the heat-emitting elements leads to an increase in deviations of heat flux from the steady-state regime and a reduction in deviations of element temperature from the steady-state regime.

Thus, the completed calculations illustrate the degree to which unsteadiness of the flow affects heat transfer, and they confirm that to obtain the most reliable information, heat-transfer characteristics should be calculated in a coupled formulation.

NOTATION

Θ_1, Θ_2, Θ_3, dimensionless temperatures of the fuel element, casing, and fluid, respectively; $X = x/d$, $Y = y/d$, $Z = z/\ell$, dimensionless coordinates; $Fo = a_3\tau/d^2$, Fourier number; $W = w_z/\bar{w}_z$, dimensionless longitudinal velocity of the coolant; γ, dimensionless amplitude of oscillations of the pressure gradient; $(dp/dz)_s$, steady-state component of the pressure gradient; ω, frequency of oscillation; d, diameter of the external surface of a casing with the thickness δ; $\bar{q}_V$, volumetric heat flux averaged over the length; $Re = \bar{w}_z d/\nu_3$, Reynolds number; 2b, distance between centers of the fuel elements; q_a, T_a, heat flux and temperature averaged over the perimeter of the casing, respectively; s, index denoting that the corresponding quantity pertains to the steady-state process; $L = d/\ell$; $K_{\lambda_i} = \lambda_i/\lambda_3$; $K_{c_i} = \rho_i c_i/(\rho_3 c_3)$; $M = \omega d^2/\nu_3$.

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